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## THE INTERACTION OF AN INCIDENT WAVE FIELD WITH A FLOATING SLENDER BODY AT ZERO SPEED

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### Abstract

A linear slender-body theory is presented for the zero-speed motions of a ship in regular free-surface waves, valid for an arbitrary angle of incidence and all wavelengths of practical interest. The velocity potential of the "radiation" problem is obtained by the superposition of a particular solution, identical to that of the short-wavelength strip theory, and a homogeneous component that accounts for the longitudinal flow interactions. A similar representation is used for the "diffraction" problem where the particular solution is equal and opposite to the incident wave velocity potential.

Computations are presented for the vertical hydrodynamic force distribution and the heave and pitch added-mass and damping coefficients, exciting forces and motions for a Series 60 hull. Comparisons are made with strip theory, a three-dimensional theory and experiments.

### 1. Introduction

In the past twenty years slender-body theory techniques have found broad applications in ship hydrodynamics. The geometrical slenderness of the ship hull, however, is insufficient to justify an asymptotic theory. The length-scales introduced by the wavelength and/or the ship forward speed need also to be considered.

The wavelengths encountered in an ambient seaway are comparable to both the transverse and the longitudinal dimensions of conventional ships. It is therefore desirable that a theory for the motions of a ship advancing in waves would embrace all wavelengths encountered in practice. Newman (1978) developed the theoretical framework of such a slender-body theory where the ship slenderness is sufficient for the asymptotic analysis involved. The theory, hereafter referred to as "uni-

fied theory", was applied to the radiation problem by Newman and Sclavounos (1980) and was extended to the diffraction problem by Sclavounos (1981).

In the present paper the radiation and diffraction problems are combined to predict the zero-speed heave and pitch motions of a ship in waves. The theoretical derivation is presented in Section 2 including the radiation problem as derived by Newman (1978) for purposes of comparison with the diffraction problem. The short-wavelength approximations of the unified theory are derived in Section 3 where existing theories for the radiation and diffraction problems are recovered. An efficient numerical technique for the solution of two-dimensional free-surface problems is described in Section 4. A hybrid integral representation valid in a domain bounded by the body section, the free surface and a circular boundary lying in the fluid domain is matched to a multipole representation valid outside the matching circle. In Section 5 numerical computations are presented for the hydrodynamic force distribution and the motions for a Series 60 ( $C_B=0.7$ ) hull and comparisons are made with existing theories and experimental data.

In what follows, a physical description of the method of solution is attempted. The ship is assumed to be slender, the fluid motion incompressible and irrotational and the ship motion amplitude small enough to validate the linear decomposition of the hydrodynamic disturbance in the radiation and the diffraction problems.

For the radiation problem, in an "inner" region close to the ship hull, the longitudinal flow gradients are small compared to the transverse ones. The field equation can therefore be reduced to the two-dimensional Laplace equation, supplemented by a two-dimensional body boundary condition and the zero-speed free-surface condition. For the diffraction problem the slenderness assumptions are invoked after the

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longitudinal component of the incident wave is factored out. The two-dimensional modified Helmholtz equation turns out to be the relevant inner-field equation with a no flux body-boundary condition and the same free-surface condition as in the radiation problem.

In an "outer" region, located at radial distances greater or equal to the ship length, the flow gradients in all directions are of comparable magnitude. The three-dimensional Laplace equation is satisfied in the fluid domain for both problems, subject to the zero-speed free-surface condition and a condition of outgoing waves at infinity. The method of matched asymptotic expansions is used to enforce the compatibility of the inner and outer solutions in an intermediate "overlap" region.

The velocity potential of the inner radiation problem involves a particular solution identical to that of the short-wavelength strip theory and is supplemented by a homogeneous solution that represents standing waves at "infinity". The corresponding particular solution of the diffraction problem is equal and opposite to the incident wave potential with the longitudinal wave component factored out. The homogeneous part again satisfies standing waves at "infinity" and is regular for all headings.

The homogeneous solutions of both the radiation and the diffraction problems are multiplied by "interaction" coefficients, functions of the longitudinal coordinate, that are determined after the matching with the corresponding outer solutions. Two integral equations are solved, in that context, for the unknown three-dimensional source distributions along the ship axis that describe the outer solutions of both problems.

In the short-wavelength limit, the interaction coefficient of the radiation problem vanishes and the classical strip-theory solution is recovered (see Ogilvie and Tuck (1969)). At the same limit, the solutions of Choo (1975) and Troesh (1976) for the diffraction problem are recovered for oblique waves. The singularity that is present in these theories for head waves is due to a non-uniformity that is present when the short-wavelength approximation preceeds the transition from oblique to head incidence. For head waves the Maruo and Sasaki (1974) theory is recovered, apart from a minor analytical difference.

## 2. The Boundary Value Problem

We introduce a Cartesian coordinate system fixed in space with the free surface taken at  $z=0$ , the ship center-plane at  $y=0$  and the positive axis pointing towards the bow. The ship is assumed to perform small oscillatory motions around its undisturbed floating position, all oscillatory quantities are expressed in complex form and the factor  $e^{i\omega t}$  is understood hereafter. Only the symmetric heave and pitch motions are considered, since, to the same degree of accuracy, the longitudinal interactions are absent from the anti-symmetric modes for which strip theory is valid for all wavelengths. More details for the analysis that follows can be found in Newman (1978) and Scavounous (1981).

For water of infinite depth the incident wave potential is given by

$$\Phi_I = \frac{igA}{\omega} \exp[vz - iv(x\cos\beta + y\sin\beta)], \quad (2.1)$$

where  $A$  is the wave amplitude,  $v=\omega^2/g$  is the wavenumber and  $\beta$  the angle of incidence, with  $\beta=180^\circ$  for head incidence.

Let  $\Phi_j$ ,  $j=3,5,7$  be the complex velocity potentials associated with the unit amplitude heave and pitch motions and the symmetric diffraction problem respectively. Under the assumptions stated in the Introduction, the three-dimensional Laplace equation is satisfied in the fluid domain

$$\nabla^2 \Phi_j = 0, \quad (2.2)$$

subject to the linear free-surface condition

$$\Phi_{jz} - v\Phi_j = 0 \quad \text{on } z = 0, \quad (2.3)$$

and the body boundary condition applied at the mean position of the ship wetted surface  $S$

$$(\vec{n} \cdot \vec{v})\Phi_j = \begin{bmatrix} i\omega n_j \\ -(\vec{n} \cdot \vec{v}) \left( \frac{ig}{\omega} e^{vz - ivx\cos\beta} \right. \\ \left. \cdot \cos(vys\sin\beta) \right), \quad j=3,5 \end{bmatrix}, \quad (2.4)$$

where  $\vec{n} = (n_1, n_2, n_3)$  is the unit normal vector pointing out of the fluid domain and  $n_5 = -xn_3 + zn_1$ . The well-posedness of the previous boundary-value problems requires that the velocity potentials  $\phi_j$  represent outgoing waves in the far field with the fluid velocity vanishing as  $z \rightarrow \infty$ .

In the next two subsections the ship slenderness is invoked to approximate the flow equations in the outer and inner regions respectively.

### 2A. Outer Region

At radial distances comparable to or greater than the ship length, the flow is insensitive to the ship hull geometry details and the velocity potential can be approximated by a line distribution of three-dimensional sources along the ship centerline, given by

$$\begin{aligned} \phi_j &= e^{iv_0 x} \psi_j(x, y, z) \\ &= \int_L^{iv_0 x} e^{iv_0 \xi} q_j(\xi) G(x-\xi, y, z) d\xi, \end{aligned} \quad (2.5)$$

where  $v$  is the characteristic wavenumber of the longitudinal distribution of the hull normal velocity ( $v_0 = 0$  for the heave and pitch motions and  $v_0 = -v \cos \theta$  for the diffraction problem);  $q_j(x)$  is the slowly varying part of the source strength distribution and  $G(x-\xi, y, z)$  is the velocity potential due to a unit pulsating source located on the ship axis at  $x=\xi$ . In order to match the outer solution (2.5) to the corresponding expression of the inner problem we need to expand (2.5) for small  $vr$ , where  $r = (y^2 + z^2)^{1/2}$ . It is convenient to take the Fourier transform of both sides of (2.5) to obtain,

$$\psi_j^*(y, z; k) = q_j^*(k) G^*(y, z; k - v_0), \quad (2.6)$$

where

$$\begin{aligned} G^*(y, z; k) &= \int_{-infty}^{infty} e^{ikx} G(x, y, z) dx \\ &= -\frac{1}{2\pi} \int_0^{infty} \frac{e^{z(u^2+k^2)^{1/2}} \cos uy}{(u^2+k^2)^{1/2}-v} du. \end{aligned} \quad (2.7)$$

For small  $vr$ ,  $v_0 r$  and  $kr$ ,  $G^*$  can be approximated in the form

$$G^*(y, z; k - v_0) = \operatorname{Re}\{G^*(y, z; -v_0)\}$$

$$- \frac{1}{2\pi} (1+vr) F^*(k - v_0) \quad (2.8)$$

with an error factor  $1+0(v^2 r^2, v_0^2 r^2, k^2 r^2)$  and

$$\begin{aligned} F^*(k) &= \ln(|v_0|/|k|) + (1 - v_0^2/v^2)^{-1/2} \\ &\cdot \cosh^{-1}(v/v_0) - |1 - k^2/v^2|^{-1/2} \\ &\cdot \left[ \begin{array}{l} \pi i + \cosh^{-1}(v/|k|) \\ -\pi + \cos^{-1}(v/|k|) \end{array} \right], \end{aligned} \quad (2.9)$$

where the upper or lower term in brackets is applicable according as  $v/|k| \gtrless 1$  and  $|v_0| \leq v$  for all cases of interest.

Expression (2.8) is a generalization of the corresponding approximation derived by Newman (1978, eq. 4.12) for the radiation problem. As  $v_0 \rightarrow 0$ , the combination of the first two terms in (2.9) reduces to  $\ln(2v/|k|)$ . Furthermore,  $G^*(y, z; 0) = R_{2D}(y, z)$  is the two-dimensional source potential that satisfies the Laplace equation, subject to the linear free-surface condition and a radiation condition of outgoing waves as  $v|y| \rightarrow \infty$ . Given that  $\operatorname{Im}(R_{2D}) = \frac{1}{2} e^{vz} \cos(vy)$  and the fact that  $e^{vz} \cos(vy) = (1+vr)[1+0(v^2 r^2)]$  in the regime where (2.8) is valid, it follows that

$$G^*(y, z; k) = R_{2D}(y, z) - \frac{1}{2\pi} (1+vr) f^*(k), \quad (2.10)$$

where

$$\begin{aligned} f^*(k) &= \ln(2v/|k|) + \pi i - |1 - k^2/v^2|^{-1/2} \\ &\cdot \left[ \begin{array}{l} \pi i + \cosh^{-1}(v/|k|) \\ -\pi + \cos^{-1}(v/|k|) \end{array} \right], \end{aligned} \quad (2.11)$$

which is identical to equation (4.12) of Newman (1978).

For the diffraction problem,  $v_o = -v\cos\beta$ , with  $G^*(y, z; -v_o) = D_{2D}$  being the corresponding two-dimensional source potential that satisfies the Helmholtz equation  $(\nabla_{2D}^2 - v_o^2)D_{2D} = 0$  and the same free-surface and radiation conditions as  $R_{2D}$ . It is essential to point out that  $\text{Im}(D_{2D}) = \frac{1}{2}\csc\beta e^{vz} \cos(vysin\beta)$  is singular for  $|\cos\beta|=1$  and, as opposed to the radiation problem, it is not present. Thus, the inner expansion of the outer solution for the diffraction problem is given by (2.6) and (2.8)-(2.9) and is regular for all headings.

In summary, the inner expansion of the outer solution transformed back in the physical x-space takes the form

$$\psi_j(x, y, z) = q_j(x) \begin{cases} R_{2D} \\ \text{Re}(D_{2D}) \end{cases} - \frac{1}{2\pi}(1+vz) \mathcal{L}(q_j), \quad (2.12)$$

where the linear operator  $\mathcal{L}$  is obtained by taking the inverse Fourier transform of (2.6). It follows that

$$\begin{aligned} \mathcal{L}(q_j) &= q_j(x) \left[ \gamma + \left\{ \begin{array}{l} \frac{\pi i}{h(\beta)} \\ h(\beta) \end{array} \right\} \right] \\ &+ \int_{-\infty}^{\infty} d\xi e^{-i\beta_o(x-\xi)} \left\{ \frac{1}{2} \text{sgn}(x-\xi) \ln(2v|x-\xi|) \right. \\ &\quad \left. + \left( \frac{d}{d\xi} + iv_o \right) q_j(\xi) - \frac{\pi v}{4} K(v(x-\xi)) q_j(\xi) \right\}, \end{aligned} \quad (2.13)$$

where  $\gamma=0.57\dots$  is the Euler constant,

$$h(\beta) = \csc\beta \cosh^{-1}(|\sec\beta|) - \ln(2|\sec\beta|) \quad (2.14)$$

$$K(x) = Y_o(x) + 2i J_o(|x|) + H_o(|x|) \quad (2.15)$$

with the upper or lower terms in brackets applying for the radiation ( $j=3, 5$  and  $v_o=0$ ) and the diffraction ( $j=7$  and  $v_o = -v\cos\beta$ ) problems respectively.

## 2B. Inner Region

At transverse distances of the order of the ship beam, the relative order of the flow gradients in the longitudinal and transverse directions are dictated by the respective order of the geometry gradients. Assuming that both the y and z coordinates are of  $O(\epsilon)$ , a coor-

dinate stretching suggests that

$$\frac{\partial}{\partial x} = O(1), \quad \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial z} = O(\epsilon^{-1}) \quad (2.16)$$

when operating on  $\psi_j$  defined in (2.5).

Applying (2.16) in (2.2)-(2.4) and using the definition of  $\psi_j$ , we obtain the two-dimensional modified Helmholtz equation

$$\psi_{jyy} + \psi_{jzz} - v_o^2 \psi_j = 0, \quad (2.17)$$

subject to the free-surface condition

$$\psi_{jz} - v\psi_j = 0 \quad \text{on } z=0, \quad (2.18)$$

and the body boundary condition

$$\frac{\partial}{\partial N} \psi_j = \begin{cases} i\omega N_j, & j=3, 5 \\ -\frac{ig}{\omega} \frac{\partial}{\partial N} \{e^{vz} \cos(vysin\beta)\}, & j=7 \end{cases}, \quad (2.19)$$

where to leading order in  $\epsilon$ ,  $\vec{n} = (0, N_2, N_3)$ , with  $N_5 = -xN_3$ .

The general solution of (2.17)-(2.19) can be obtained in the form

$$\psi_j = \psi_{jp} + C_j(x) \psi_{jH}, \quad (2.20)$$

where  $\psi_{jp}$  is a particular and  $\psi_{jH}$  a homogeneous solution of (2.17)-(2.19) respectively. The "interaction" coefficient  $C_j(x)$  is presently an arbitrary function of  $x$  that will be determined from the matching with the outer solution. No radiation condition needs to be satisfied in the inner region since the outer solution already represents outgoing waves at infinity.

Following Newman (1978), a particular solution of the radiation problem ( $j=3, 5$ ,  $v=0$ ) is the short-wavelength strip theory potential  $\phi_j$  that satisfies outgoing waves as  $v|y|\rightarrow\infty$ . The homogeneous solution can be physically regarded as the interaction of two incident symmetric waves with the ship section. The pure imaginary form of the body boundary condition suggests that  $\phi_j + \bar{\phi}_j$  is a homogeneous solution, where the overbar stands for the complex conjugate of the quantity involved. Thus,

$$\phi_j = \psi_j = \phi_j + C_j(x)(\phi_j + \bar{\phi}_j), \quad j=3, 5 \quad (2.21)$$

with  $\phi_5 = -x\phi_3$ . In the overlap region, at transverse distances large compared to the ship beam but still small compared to its length as  $\epsilon \rightarrow 0$ ,  $\phi_j$  can be expressed by their effective source strengths, in the form

$$\phi_j = \sigma_j R_{2D}(y, z), \quad (2.22)$$

where  $R_{2D}$  is the two-dimensional source potential that satisfies outgoing waves as  $v|y| \rightarrow \infty$ . We may thus approximate  $\phi_j$  in the overlap region as follows,

$$\begin{aligned} \phi_j &= \{\sigma_j + C_j(x)(\sigma_j + \bar{\sigma}_j)\} R_{2D} \\ &\quad - 2i C_j(x) \bar{\sigma}_j \operatorname{Im}(R_{2D}). \end{aligned} \quad (2.23)$$

If we now set,  $\operatorname{Im}(R_{2D}) = \frac{1}{\pi}(1+vz)\{1+0(v^2 r^2)\}$ , the outer expansion of the inner solution becomes

$$\begin{aligned} \phi_j &= \{\sigma_j + C_j(x)(\sigma_j + \bar{\sigma}_j)\} R_{2D} \\ &\quad - i C_j(x) \bar{\sigma}_j (1+vz). \end{aligned} \quad (2.24)$$

Comparing (2.12) to (2.24) the matching conditions for the radiation problem are,

$$\sigma_j + C_j(x)(\sigma_j + \bar{\sigma}_j) = q_j \quad (2.25)$$

$$i C_j \bar{\sigma}_j = \frac{1}{2\pi} \mathcal{L}(q_j). \quad (2.26)$$

After eliminating  $C_j$  from (2.25)-(2.26), the outer source strength  $q_j$  is determined from the integral equation

$$\begin{aligned} q_j(x) &= \frac{1}{2\pi i} (\sigma_j / \bar{\sigma}_j + 1) \mathcal{L}(q_j) \\ &= \sigma_j(x) \quad j=3, 5, \end{aligned} \quad (2.27)$$

where  $\mathcal{L}(q_j)$  is given by (2.13) with  $v_0 = 0$ .

A particular solution for the diffraction problem follows easily from the body boundary condition (2.19) and is equal and opposite to the symmetric part of the incident wave potential

$$\psi_{7p} = -\frac{iq}{\omega} e^{vz} \cos(vysin\beta). \quad (2.28)$$

The head sea limit of  $\psi_{7p}$  is the leading order solution in the short-wavelength theory of Faltinsen (1971).

The homogeneous solution again results from the interaction of two waves of unit amplitude and equal phase incident from opposite directions upon

the ship section, and is obtained in the form

$$\psi_{7H} = \frac{iq}{\omega} e^{vz} \cos(vysin\beta) + \psi_D(y, z), \quad (2.29)$$

where  $\psi_D$  is constructed by using the two-dimensional source potential  $\operatorname{Re}(D_{2D})$  that represents standing waves as  $v|y| \rightarrow \infty$  and is regular for all headings. Thus,

$$\begin{aligned} \psi_7 &= -\frac{iq}{\omega} e^{vz} \cos(vysin\beta) [1 - C_7(x)] \\ &\quad + C_7(x) \psi_D(y, z). \end{aligned} \quad (2.30)$$

In the overlap region,  $\psi_7$  can be approximated in the form

$$\begin{aligned} \psi_7 &= -\frac{iq}{\omega} [1 - C_7(x)] (1 + vz) \\ &\quad + C_7(x) \sigma_7(x) \operatorname{Re}(D_{2D}). \end{aligned} \quad (2.31)$$

The matching requirements again follow easily from (2.12) and (2.31),

$$C_7 \sigma_7 = q_7 \quad (2.32)$$

$$\frac{iq}{\omega} (1 - C_7) = \frac{1}{2\pi} \mathcal{L}(q_7). \quad (2.33)$$

The elimination of  $C_7$  from (2.32)-(2.33) results in the following integral equation for  $q_7$ ,

$$q_7(x) - \frac{iq}{2\pi} \sigma_7 \mathcal{L}(q_7) = \sigma_7, \quad (2.34)$$

where  $\mathcal{L}(q_7)$  is determined by (2.13) with  $v_0 = -\sqrt{v} \cos\beta$ .

The solution of equations (2.27) and (2.34) determines the outer source strength  $q_j$ , and the complete inner solutions follow in the form

$$\phi_j = \phi_j + \frac{q_j - \sigma_j}{\sigma_j + \bar{\sigma}_j} (\phi_j + \bar{\phi}_j) \quad j=3, 5 \quad (2.35)$$

for the radiation problem, and

$$\begin{aligned} \psi_7 &= \frac{iq}{\omega} e^{vz} \cos(vysin\beta) (q_7 / \sigma_7 - 1) \\ &\quad + \psi_D q_7 / \sigma_7 \end{aligned} \quad (2.36)$$

for the diffraction problem.

### 3. Short-Wavelength Approximations

For wavelengths comparable to the ship transverse dimensions ( $vL=0(\epsilon^{-1})$  as  $\epsilon \rightarrow 0$ ), it is possible to approximate the linear operator  $\mathcal{L}(q)$  defined in (2.13) using the techniques of asymptotic analysis. For the diffraction problem and  $-1 < \cos\beta \leq 0$ , it follows that

$$\mathcal{L}(q_7) = \frac{1}{2}(1-i)(\pi v^{\frac{1}{2}}) \int_x^{L/2} q_7(\xi) \frac{e^{-iv(\xi-x)(1+\cos\beta)}}{(\xi-x)^{\frac{1}{2}}} d\xi$$

$$-i(\csc\beta - \frac{1}{2}|\sec\frac{\beta}{2}|)q_7(x) + O(v^{-\frac{1}{2}}). \quad (3.1)$$

The details of the derivation can be found in Sciauounos (1981). The corresponding approximation for the radiation problem follows from (2.13) and (3.1) with  $\cos\beta=0$ , in the form

$$\mathcal{L}(q_j) = \frac{1}{2}(1-i)(\pi v)^{\frac{1}{2}} \int_x^{L/2} q_j(\xi) \frac{e^{-iv(\xi-x)}}{(\xi-x)^{\frac{1}{2}}} d\xi + \frac{\sqrt{2}}{2} q_j(x) + O(v^{-\frac{1}{2}}). \quad (3.2)$$

Proceeding with the radiation problem, the integral in (3.2) can be further approximated for large  $v$  in the form

$$\int_x^{L/2} q_j(\xi) \frac{e^{-iv(\xi-x)}}{(\xi-x)^{\frac{1}{2}}} d\xi = -\left(\frac{\pi}{v}\right)^{\frac{1}{2}} e^{-\pi i/4} q_j(x). \quad (3.3)$$

The substitution of (3.3) in (3.2) cancels out the first two terms, with the remaining part being of  $O(v^{-\frac{1}{2}})$ . Thus for short wavelengths

$$q_j(x) = \sigma_j(x) + O(v^{-\frac{1}{2}}), \quad (3.4)$$

and strip theory is recovered.

An approximation similar to (3.3) applied to (3.1) gives

$$\mathcal{L}(q_7) = -i \csc\beta q_7(x) + O(v^{-\frac{1}{2}}). \quad (3.5)$$

The approximation (3.5) is singular for head waves due to a nonuniformity that is present in the asymptotic approximation of (3.1) when  $1+\cos\beta=0$ . Substituting (3.5) in (2.34) we obtain an algebraic equation for  $q_7(x)$  with a solution

$$q_7 = \sigma_7 / (1 + \frac{w}{2g} \sigma_7 \csc\beta). \quad (3.6)$$

Substituting (2.33) and (2.32) in (2.30) and using (3.5), we may write the outer expansion of the inner solution in the form

$$\psi_7 = \frac{1}{2} i \csc\beta q_7(x) (1+vz) + q_7(x) \operatorname{Re}(D_{2D}) \\ = q_7(x) D_{2D}, \quad (3.7)$$

where  $1+vz = e^{vz} \cos(v \sin\beta) [1+O(v^2 r^2)]$  in the overlap region. The two-dimensional source potential  $D_{2D}$  satisfies outgoing waves as  $v|y| \rightarrow \infty$  is singular for head waves and corresponds to the Green function used in the short-wavelength strip theory derived by Choo (1975) and Troesh (1976).

In the head-sea case ( $\beta=180^\circ$ ), (3.1) reduces to (originally due to Faltinsen (1971))

$$\mathcal{L}(q_7) = \frac{1}{2} (1-i)(\pi v)^{\frac{1}{2}} \int_x^{L/2} \frac{q_7(\xi)}{(\xi-x)^{\frac{1}{2}}} d\xi, \quad (3.8)$$

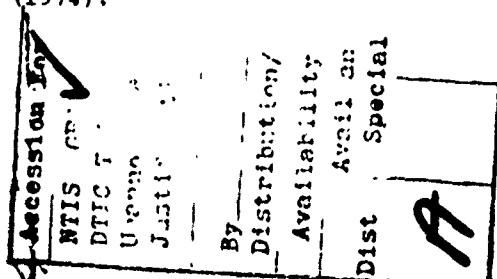
and the Maruo and Sasaki (1974) theory is obtained apart from two analytical differences; the  $\frac{1}{2}$  factor is missing in their theory and an additional  $-i q_7(x)/2$  term appears in their equation which corresponds to (3.8) that should not be present in a consistent short-wavelength approximation.

### 4. The Two-Dimensional Problem

In this section a numerical scheme for the solution of two-dimensional free-surface wave problems in water of infinite depth is described.

The application of spectral techniques to the solution of free-surface boundary-value problems is in general restricted to simple body profiles. Ursell (1949) obtained the solution of the heaving problem of a circular section in water of infinite depth by using a sequence of singularities located at the origin of the axes. The extension, however, of this method to more general body sections has not been yet formally established.

An efficient numerical technique to treat arbitrary body and bottom geometries has been developed by Yeung [Bai and Yeung (1974)]. In an inner region of changing topography, a hybrid integral equation is obtained by applying Green's theorem and using the fundamental logarithmic singularity as the relevant Green function. This representation is then matched to an eigenfunction expansion in an outer region of constant depth. Liapis and Faltinsen (1980) extended this method to the case where the Helmholtz equation is satisfied in the fluid domain. A comprehensive survey of existing techniques for the solution of two-dimensional free-surface wave problems is given by Wehausen (1974).



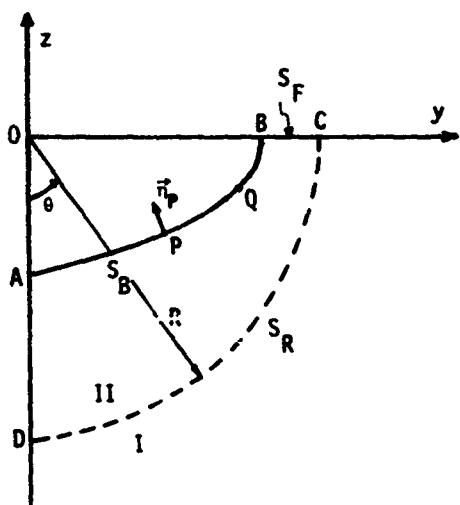


Figure 1.

The present approach is a coupling of a multipole expansion and a hybrid integral representation, valid respectively in the domains I and II of Figure 1, separated by a matching circle  $S_F$ . The water depth is assumed infinite and the body section symmetric with respect to the  $y=0$  axis. The Helmholtz equation is in general satisfied in the fluid domain

$$\nabla^2 \phi - \ell^2 \phi = 0, \quad (4.1)$$

subject to the free-surface condition

$$\phi_z - v \phi = 0 \quad \text{on } z=0, \quad (4.2)$$

to a general body-boundary condition

$$(\vec{n}_p \cdot \nabla) \phi = v(P) \quad \text{on } S_B, \quad (4.3)$$

and to a condition of standing or outgoing waves as  $v|y| \rightarrow \infty$ . For all cases of interest  $0 < \ell \leq v$ .

Assuming a symmetric flow with respect to the  $y=0$  axis, we may write the velocity potential in region I in the form [Ursell (1949, 1968)]

$$\phi^I = a_0 W_0(r, \theta) + \sum_{m=1}^{\infty} a_m W_m(r, \theta), \quad (4.4)$$

where  $a_0$  is the effective source strength,  $W_0$  is the relevant wave source potential due to a unit source located at the origin, and  $W_m(r, \theta)$  are wave-free multipoles. For  $\ell = v|\cos\theta| > 0$ ,  $W_0 = R_0 D_{2D}$  with  $D_{2D}$  defined in Section 2, and

$$\begin{aligned} W_m(r, \theta) = & K_{2m-2}(\ell r) \cos(2m-2)\theta \\ & + K_{2m}(\ell r) \cos 2m\theta \\ & + 2|\sec\theta| K_{2m-1}(\ell r) \cos(2m-1)\theta \end{aligned} \quad (4.5)$$

where  $K_n(x)$  is the modified Bessel function of order  $n$  defined in Abramowitz and Stegun (1964). For  $\ell=0$ ,  $W_0 = R_{2D}$  with  $R_{2D}$  defined in Section 2, and

$$W_m(r, \theta) = \frac{\cos 2m\theta}{r^{2m}} + \frac{v}{2m-1} \frac{\cos(2m-1)\theta}{r^{2m-1}} \quad (4.6)$$

Applying Green's theorem in region II (ABCDA), using the symmetry of the flow and the free surface condition (4.2), we obtain

$$\begin{aligned} & -\frac{1}{2} \phi^{II}(Q) + \int_{S_B \cup S_F} ds(P) \phi^{II}(P) (\vec{n}_p \cdot \nabla_p) F(P, Q) \\ & + \int_{S_F} ds(P) \phi^{II}(P) [(\vec{n}_p \cdot \nabla_p) - v] F(P, Q) \\ & - \int_{S_R} ds(P) F(P, Q) (\vec{n}_p \cdot \nabla_p) \phi^{II}(P) \\ & = \int_{S_B} ds(P) F(P, Q) v(P), \end{aligned} \quad (4.7)$$

where  $\vec{n}_p$  is the unit normal vector pointing out of domain II and

$$F(P, Q) = G(P, Q^+) + G(P, Q^-), \quad (4.8)$$

with  $Q^+ \equiv Q$  lying on  $S_B \cup S_F \cup S_R$  and  $Q^-$  being the symmetric of  $Q$  with respect to the  $y=0$  axis;  $G(P, Q)$  is the velocity potential at  $P$  due to a fundamental unit source located at  $Q$ , given by

$$G(P, Q) = \begin{cases} \frac{1}{2\pi} \log(r_{PQ}) & , \ell=0 \\ -\frac{1}{2\pi} K_0(\ell r_{PQ}) & , \ell>0 \end{cases}, \quad (4.9)$$

where  $K_0(x)$  is the modified Bessel function of zeroth order.

A grid is selected along the contours  $S_B$ ,  $S_F$  and  $S_R$  with the velocity potential  $\phi$  assumed constant along the segments joining two consecutive grid points. Let  $N_B$ ,  $N_F$  and  $N_R$  be the number of segments along  $S_B$ ,  $S_F$  and  $S_R$  respectively. Taking the segment lengths to be of equal length along the matching circle and keeping  $N_R-1$  terms in the series (4.4), the velocity potential  $\phi^I$  and its  $r$ -derivative  $\phi_r^I$  at the segment midpoints can be written in a matrix form

$$\phi^I = A \vec{a} \quad (4.10)$$

$$\phi_r^I = B \vec{a}, \quad (4.11)$$

where the column vectors  $\phi^I$ ,  $\vec{a}$  and the matrixes  $A$  and  $B$  are defined as follows

$$\phi^I = \{\phi^I(R, \theta_1), \dots, \phi^I(R, \theta_{N_R})\}^T \quad (4.12)$$

$$\vec{a} = \{a_0, \dots, a_{N_R-1}\}^T \quad (4.13)$$

and

$$A = A_{ij} = W_{j-1}(R, \theta_i), B = \frac{\partial A}{\partial r} \Big|_{r=R} \quad (4.14)$$

where  $(R, \theta_i)$ ,  $i=1, \dots, N_R$  are the polar coordinates of the segment midpoints along the discretized circle  $S_p$ , with  $\theta_i$  decreasing to zero as  $i$  increases.

If we left-multiply both sides of equation (4.10) by  $BA^{-1}$ , we obtain a relation between the velocity potential  $\phi^I$  and the normal velocity  $\phi_r^I$ , in the form

$$\phi_r^I = B A^{-1} \phi^I. \quad (4.15)$$

We next proceed to the discretization of equation (4.7). The velocity potential at the midpoint of the  $i$ -th segment is denoted by  $\phi_j^{II}$ , where the indexing starts from the point A of Fig. 1. It follows that

$$\sum_{j=1}^{N_T} (C_{ij} - \frac{1}{2} \delta_{ij}) \phi_j^{II} - \sum_{j=N_F+1}^{N_T} S_{ij} \phi_{rj}^{II} = \sum_{j=1}^{N_B} S_{ij} v_j, \quad (4.16)$$

where  $N_T = N_B + N_F + N_R$ ,  $v_j$  is the body boundary normal velocity,  $\delta_{ij}$  the Kronecker delta and

$$C_{ij} = \begin{cases} D_{ij} & j=1, \dots, N_B \text{ and } j=N_F+1, \dots, N_T \\ D_{ij} - v S_{ij} & j=N_B+1, \dots, N_F \end{cases} \quad (4.17)$$

with  $i=1, \dots, N_T$ . The influence matrixes  $S_{ij}$ ,  $D_{ij}$   $i, j = 1, \dots, N_T$  are defined as follows

$$S_{ij} = \begin{cases} ds(P) F(P, Q_i) & \\ S_j & \end{cases} \quad (4.18)$$

$$D_{ij} = \begin{cases} ds(P) (\vec{n}_P \cdot \nabla_P) F(P, Q_i), & \\ S_j & \end{cases} \quad (4.19)$$

where,  $F(P, Q)$  is defined in (4.8)-(4.9), the point  $Q_i$  is located at the midpoint of the  $i$ -th segment and the integrations in (4.18)-(4.19) are performed over the  $j$ -th segment  $S_j$ .

The matching conditions of continuous pressure and normal velocity across the matching circle  $S_R$  can be expressed in the form

$$\phi^I(R, \theta_i) = \phi_{i+N}^{II} \quad (4.20)$$

$$\phi_r^I(R, \theta_i) = \phi_{r, i+N}^{II}, \quad (4.21)$$

where  $i=1, \dots, N_R$  and  $N=N_B+N_F$ . Using (4.20)-(4.21) and (4.15) in the second sum of (4.16), we end up with a matrix equation for  $\phi_j^{II}$ ,  $j=1, \dots, N_T$

$$(C - \frac{1}{2} I - E) \phi^{II} = S \vec{v}, \quad (4.22)$$

where,

$$\phi^{II} = \{\phi_i^{II}\}^T \quad i=1, \dots, N_T \quad (4.23)$$

$$\vec{v} = \{v_i, 0, \dots, 0\}^T \quad i=1, \dots, N_B \quad (4.24)$$

$$\left. \begin{array}{l} I_{ij} = \delta_{ij} \\ S = S_{ij} \end{array} \right\} \quad i, j = 1, \dots, N_T \quad (4.25)$$

$$E = E_{ij} = \begin{bmatrix} 0 & & & \\ N_T & \sum_{k=N+1}^{N_T} s_{ik} \sum_{m=1}^{N_R} B_{k-N,m} A^{-1} & & \\ & & m, j-N, & \\ & & & j=1, \dots, N \\ & & & \\ & & & j=N+1, \dots, N_T \end{bmatrix} \quad (4.26)$$

with  $i=1, \dots, N_T$  and  $A=A_{ij}$ ,  $B=B_{ij}$ ,  $i, j=1, \dots, N_T$  defined in (4.14).

The velocity potential  $\phi$  along the contours  $S_B$ ,  $S_F$  and  $S_R$  is determined from the solution of the system of equations (4.22). The effective source strength  $a_{ij}$  in (4.4) is obtained by left-multiplying (4.10) by  $A^{-1}$ .

The present scheme is free from irregular frequencies. Furthermore, only one evaluation of the wave-source potential  $W_0$  is necessary. This reduces substantially the total computation time required, in comparison to the alternative approach of a wave-source distribution on the body boundary, especially when  $\ell > 0$ .

The described technique has been applied for  $\ell = v |\cos \beta| > 0$  in the context of the diffraction problem. Selecting, the matching circle radius  $R$  10% greater than the body-section maximum radius and 15, 10 and 15 segments on  $S_B$ ,  $S_F$  and  $S_R$  respectively, proved to be sufficient for  $0 < \frac{vB}{2} \leq \pi$ , where  $B$  is the beam of the body section. The same estimates are expected to hold in the special case of  $\ell=0$ . For this paper, the two-dimensional solutions of the radiation problem are obtained by using a computer program due to Yeung (1975), based on his hybrid integral equation technique.

## 5. Hydrodynamic Forces and Motions

The quantities of interest for the evaluation of the ship motions are the hydrodynamic pressure force and moment acting on the ship in the radiation and the diffraction problems. For a ship undergoing a steady-state small amplitude oscillatory heave and pitch motion in an otherwise calm free surface, the resulting complex force amplitude can be written in the form

$$H_i = \sum_{j=3,5} n_j t_{ij}, \quad i=3,5, \quad (5.1)$$

where  $n_j$  is the complex amplitude of the ship's displacement and

$$t_{ij} = \omega^2 a_{ij} - i\omega b_{ij} - c_{ij}. \quad (5.2)$$

The coefficients  $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$  are real and correspond respectively to the force components in phase with the acceleration, velocity and static displacement of the ship. The added-mass ( $a_{ij}$ ) and damping ( $b_{ij}$ ) coefficients can be derived from the inner velocity potential (2.21) of the radiation problem by integrating the linearized pressure obtained from Bernoulli's equation over the ship wetted surface. It follows that

$$\begin{aligned} \omega^2 a_{ij} - i\omega b_{ij} &= -i\omega \rho \iint n_i \phi_j ds \\ &- i\omega \rho \iint n_i C_j(x) (\phi_j + \bar{\phi}_j) ds, \end{aligned} \quad (5.3)$$

where all the quantities are defined in Section 2B.

The first integral in (5.3) corresponds to the strip-theory contribution. The second integral is the correction due to the three-dimensional interaction effects introduced by the unified theory. As  $v \rightarrow \infty$ , it follows from (2.25) and (3.4) that  $C_j(x) \rightarrow 0$  and the strip-theory is recovered.

Proceeding to the diffraction problem, the resulting complex amplitude of the wave exciting force is simply

$$F_i = A X_i, \quad i=3,5, \quad (5.4)$$

where  $A$  is the amplitude of the incident wave. Using the incident-wave potential (2.1) with  $A=1$  and the diffraction potential  $\phi_7$ , the unit-amplitude exciting force  $X_i$  can be expressed in the form

$$X_i = -i\omega \rho \iint (\phi_I + \phi_7) n_i ds. \quad (5.5)$$

An alternative expression can be obtained by using the Haskind relations. Combining the body-boundary condition (2.4) for the radiation problem and applying Green's theorem in (5.5), we obtain

$$X_i = -\rho \iint (i\omega n_i \phi_I - \phi_i \frac{\partial \phi_I}{\partial n}) ds, \quad (5.6)$$

where  $\phi_i$  is the radiation potential due to the  $i$ -th mode of oscillation. With the velocity potentials  $\phi_7$  and  $\phi_i$  assumed known, no assumptions are involved

regarding the wavelength or the body geometry in (5.5) and (5.6). The advantage however in using (5.6) is that the solution of the diffraction problem is not needed for the evaluation of the exciting force and moment of a body in waves.

For a slender ship, the inner diffraction velocity potential defined in (2.5) and (2.36) is used together with (2.1) to give

$$x_i = -i\omega \int e^{-i\omega x \cos \beta} C_7(x) \psi_{7H} n_i ds \quad (5.7)$$

where  $C_7$  and  $\psi_{7H}$  are defined in Section 2B. The corresponding expression for (5.6) is obtained by substituting the unified-theory radiation potential (2.21) in (5.6). It follows that

$$x_i = -\rho \int \left( i\omega n_i \phi_I - \phi_i \frac{\partial \phi_I}{\partial n} \right) ds + \rho \int \int C_i(x) (\phi_i + \bar{\phi}_i) \frac{\partial \phi_I}{\partial n} ds. \quad (5.8)$$

As  $\nu \rightarrow \infty$ , the interaction coefficient  $C_i(x) \rightarrow 0$  and expression (5.8) reduces to the exciting force obtained in the strip theory of Salvesen, Tuck and Faltinsen (1970). The short-wavelength limit of (5.7), however, does not lead to the same result since the slender-body approximations involved in (5.7) and (5.8) are different.

The relative error involved in the inner diffraction problem is a factor  $1+0(\epsilon^2 \nu \cos \beta)$ . The corresponding error factor for the radiation problem is  $1+0(\epsilon^2)$  for all wavelengths. It is thus expected that, unlike the radiation velocity potential and the exciting force (5.8), the accuracy of the diffraction potential  $\psi_{7H}$  and the corresponding exciting force (5.7) decreases with decreasing wavelength. This is supported by the comparison of the exciting force and moment predicted by (5.7) and (5.8) with an exact three-dimensional theory.

The equations of motion follow by equating the inertia forces to the sum of the pressure forces and the ship weight. Combining (5.1) and (5.4) and taking the origin of the coordinates on the ship centerline and above the center of gravity, the equations of the heave and pitch motions follow in the form

$$\sum_{j=3,5} n_j t_{ij} + A X_i = -\omega^2 \sum_{j=3,5} M_{ij} n_j, \quad (5.9)$$

where  $M_{33} = \rho V$  is the ship mass,  $M_{55}$  is the ship moment of inertia with respect to the  $y$ -axis and  $M_{35} = M_{53} = 0$  due to the special choice of the coordinate system. Substituting (5.2) in (5.9) and rearranging terms we get

$$\sum_{j=3,5} n_j \{ -\omega^2 (M_{ij} + a_{ij}) + i\omega b_{ij} + C_{ij} \} = A X_i, \quad i=3,5. \quad (5.10)$$

This is a system of linear equations that can be easily solved for the complex amplitudes  $n_j$ ,  $j=3,5$ .

Numerical computations of the hydrodynamic forces and motions were performed for a Series 60 hull ( $C_B = 0.7$ , parent form). The Salvesen, et al. (1970) strip-theory results are also shown together with an exact three-dimensional numerical solution by Inglis (1980) and experimental data, where available.

The results for the radiation problem are shown in Figures 2 and 3, and are compared to experimental data of Gerritsma (1966). The agreement between the unified theory and experiments is very good both for the hydrodynamic force distribution (Figure 2) and the added-mass and damping coefficients (Figure 3 adapted from Newman and Sclavounos (1980)), and indicates a notable improvement over strip theory. The deviation of  $a_{55}$  from the experiments that occurs at low frequencies is supported by the excellent agreement between the unified theory and the exact three-dimensional solution of Inglis (1980) for all frequencies.

The slowly varying sectional hydrodynamic force distribution for the diffraction problem is given by

$$F'_3(x) = -i\omega \rho A C_7(x) \int_C \psi_{7H} n_3 dl. \quad (5.11)$$

The corresponding expression for the Salvesen et al. (1970) strip theory is obtained from (5.8) with  $C_i(x) = 0$ , in the form

$$F'_3(x) = -\frac{i \rho g A}{\omega} \int_C (i\omega N_3 - \phi_3 \frac{\partial}{\partial N}) e^{i\omega z} \cos(\nu y \sin \beta) dl, \quad (5.12)$$

where all the quantities in (5.11) and (5.12) are defined in Section 2B. The results for  $F'_3$  and the amplitudes of the exciting force and moment shown in Figures 4 and 5 respectively, indicate that the strip-theory predictions are in general higher than those of the

unified theory.

The two expressions of the exciting force and moment (5.7) and (5.8), derived respectively by pressure integration and using the Haskind relations, compare well with each other, being in closer agreement for smaller values of  $v|\cos\beta|$ . For head waves, both agree well with the experimental data of Vugts (1971) and the three-dimensional solution, with a relatively more favorable agreement of (5.8) with the theory of Inglis (1980).

Finally, the motions of the ship free to heave and pitch are presented in Figure 6, where comparisons are made with three different sets of experimental data due to Nakamura (1966), Shintani (1966) and Yamanouchi and Ando (1966). For the model tested, the center of gravity is taken at the origin of the coordinates and the longitudinal radius of gyration is taken equal to  $1/4$  of the ship's length  $L$ .

All theories agree well in general with the experiments, except for short waves at  $\beta=180^\circ$ . The conclusions drawn for the exciting forces and moments predicted by (5.7) and (5.8) remain unchanged for the respective ship motion amplitude predictions, since the added-mass and damping coefficients are the same in the two versions.

Before comparing the motion amplitudes predicted by strip theory and the rest of the theories, it is interesting to make the following observations; for the radiation problem, strip theory underpredicts  $a_{33}$  and  $a_{55}$ , and overpredicts  $b_{33}$  and  $b_{55}$  for moderate to low frequencies, being in good agreement for high frequencies. Both these effects contribute to an overprediction of the modulus of the transfer function of the left-hand side of equation (5.10). The effects of the cross-coupling coefficients are taken into account in the computations but are relatively unimportant and are neglected in the present discussion.

For the diffraction problem, strip theory again overpredicts the exciting force and moment amplitude and since the motion amplitudes are the ratios of the quantities discussed, strip theory is in general in good agreement with the rest of the theories. The theoretical predictions of the phases agree well in general, with the exception of the heave phase at  $\beta=135^\circ$  and a departure of the three-dimensional theory for the pitch phase at  $\beta=180^\circ$ .

In conclusion, the unified theory is in excellent agreement with the exact three-dimensional theory, and both agree very well with the experimental

data. The strip theory predicts well the motion amplitudes and phases but is in less satisfactory agreement with the other theories and experiments for the hydrodynamic coefficients and the exciting forces.

The integral equations (2.27) and (2.34) are solved by iteration and the solution obtained in this manner has been checked against an independent matrix inversion solution. The computation time required for this task is minimal, leaving the solution of the two-dimensional problems as the main computational effort involved in the unified theory.

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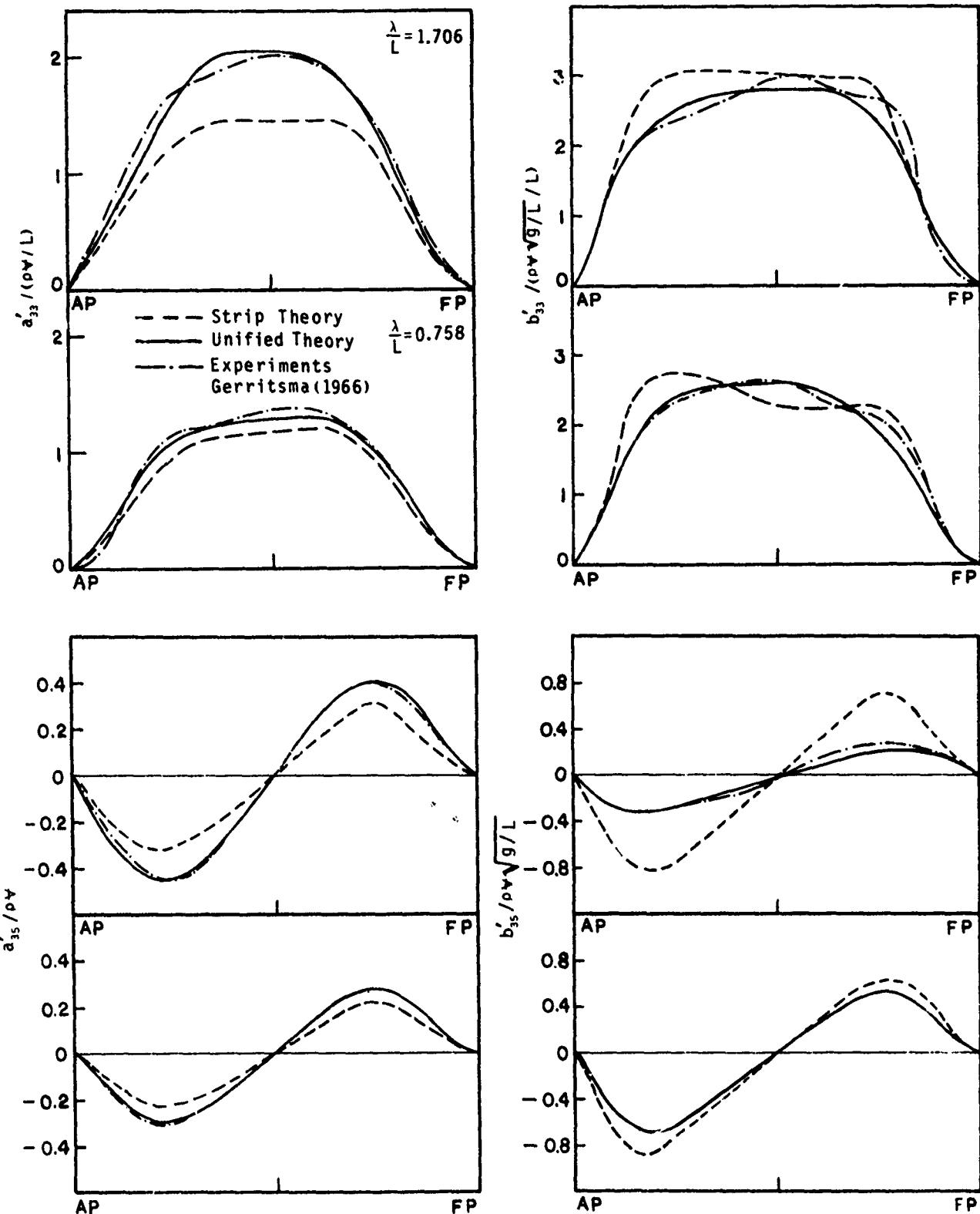


Figure 2 - Longitudinal distribution of the added-mass and damping sectional hydrodynamic force for a Series 60 hull ( $C_B = 0.7$ ) at  $\lambda/L = 1.706, 0.758$  and  $U=0$ .

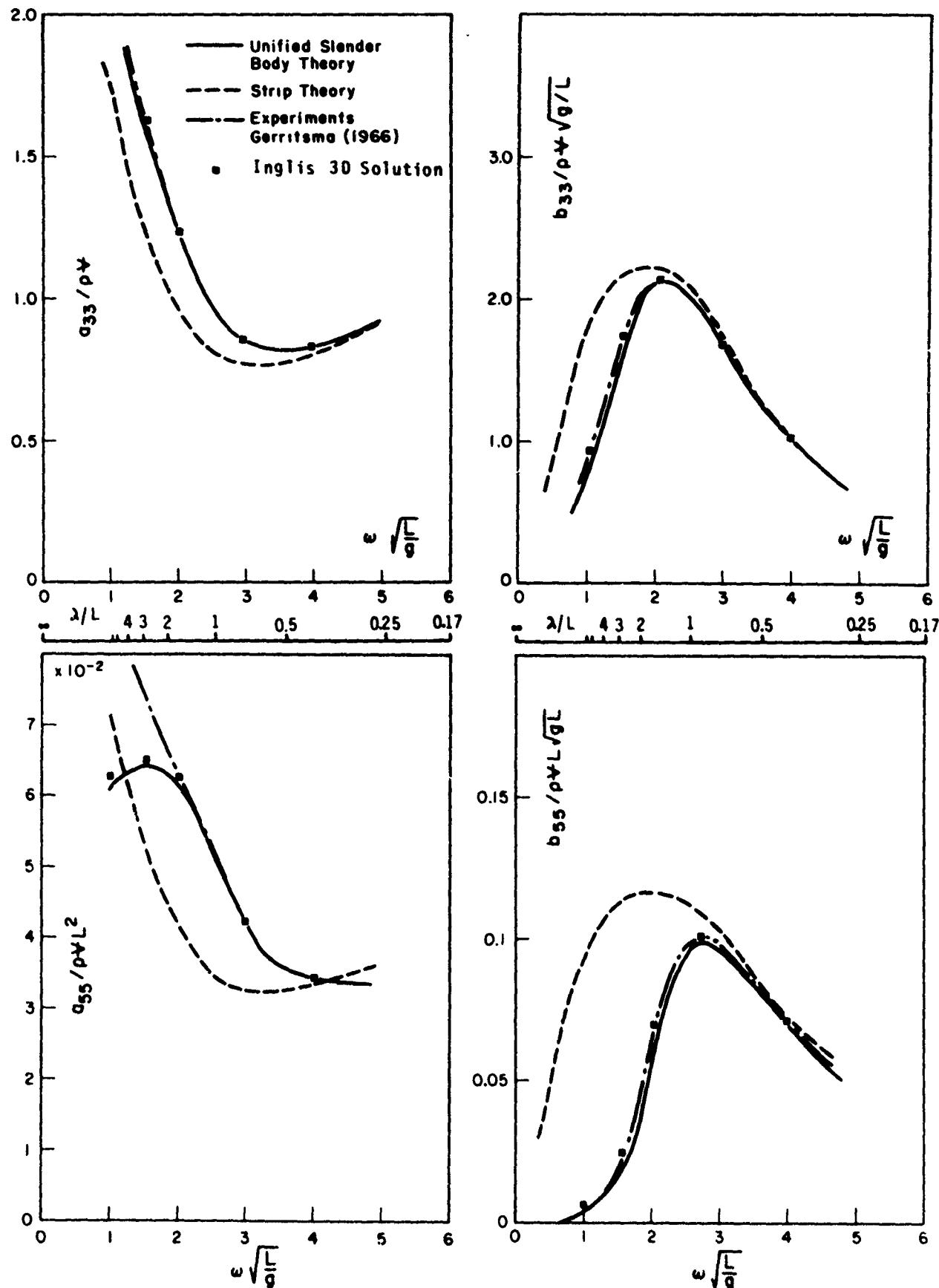


Figure 3 - Heave and pitch added-mass and damping coefficients for a Series 60 hull ( $C_B=0.7$ ) at  $U=0$  ( Adapted from Newman and Sclavounos(1980)).

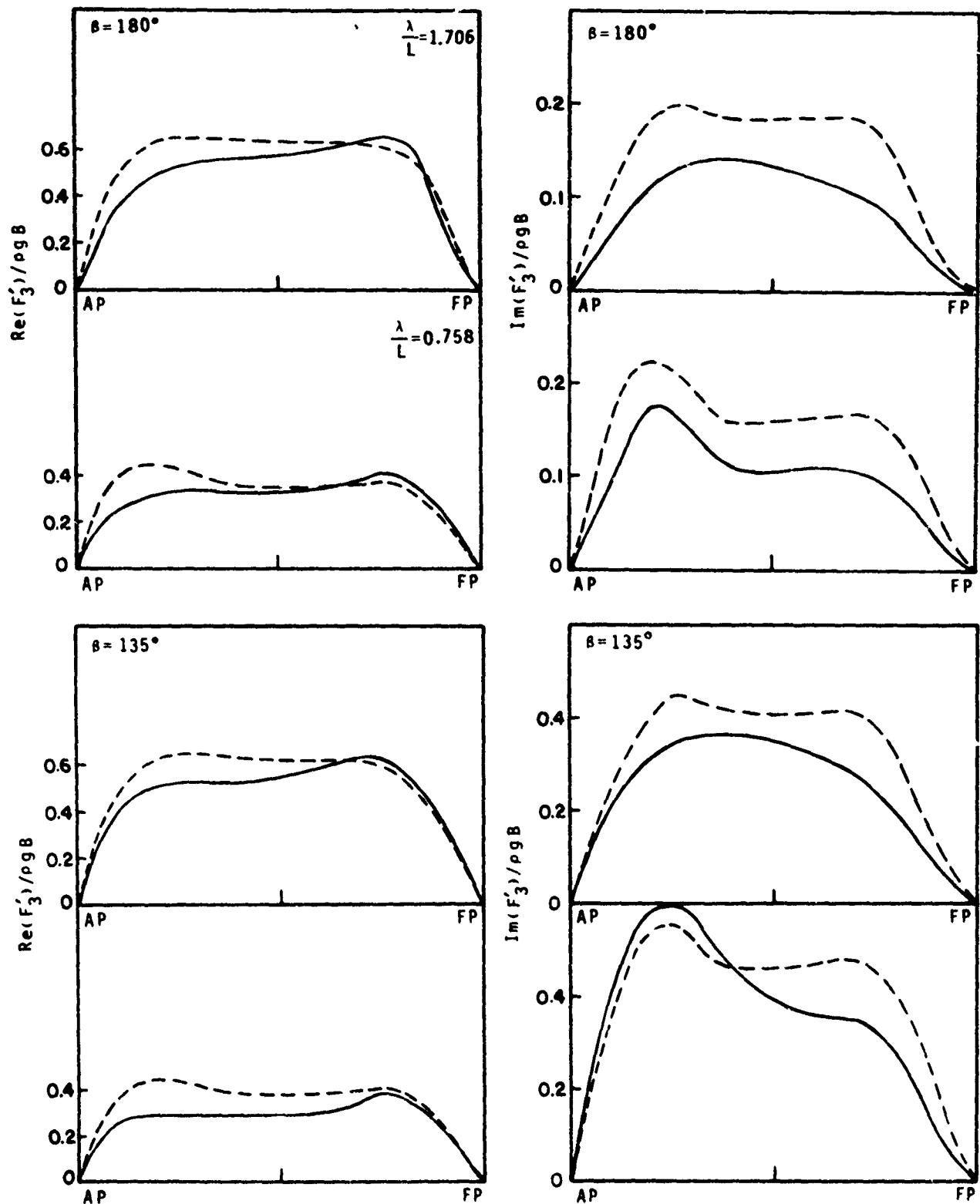


Figure 4 - Longitudinal distribution of the real and imaginary part of the sectional exciting force (with the  $\exp(i\pi x \cos\beta)$  component factored out) for a Series 60 hull ( $C_B = 0.7$ ) fixed in head ( $\beta = 180^\circ$ ) and bow ( $\beta = 135^\circ$ ) waves at  $\lambda/L = 1.706$ ,  $0.758$  and  $U=0$ .

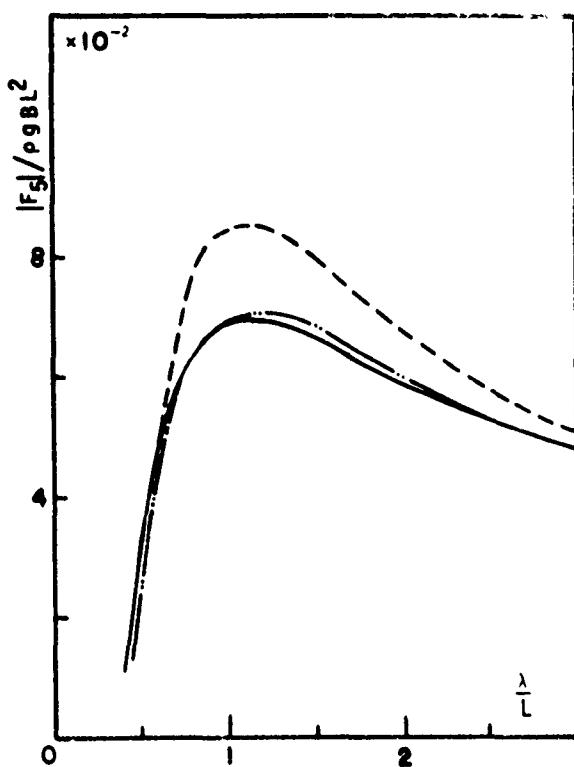
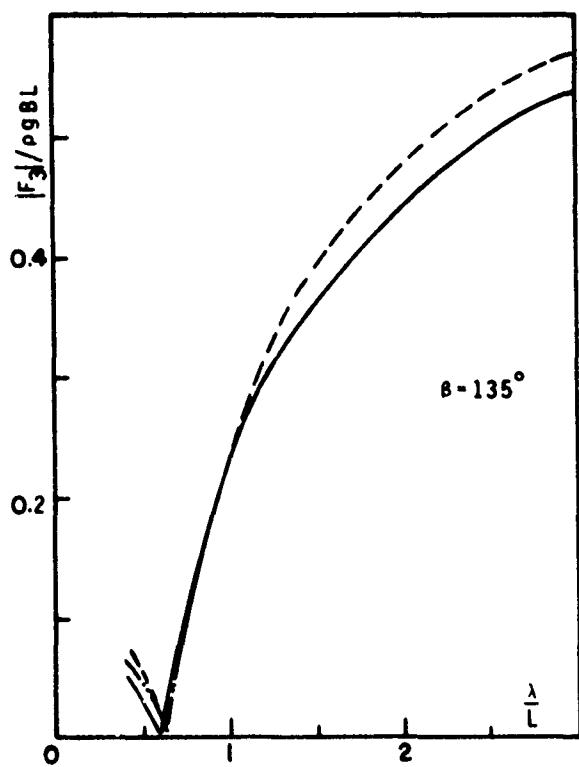
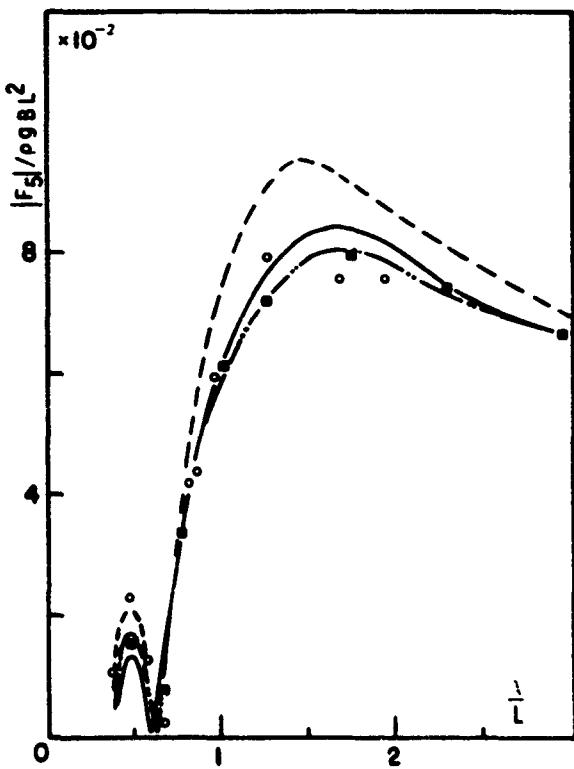
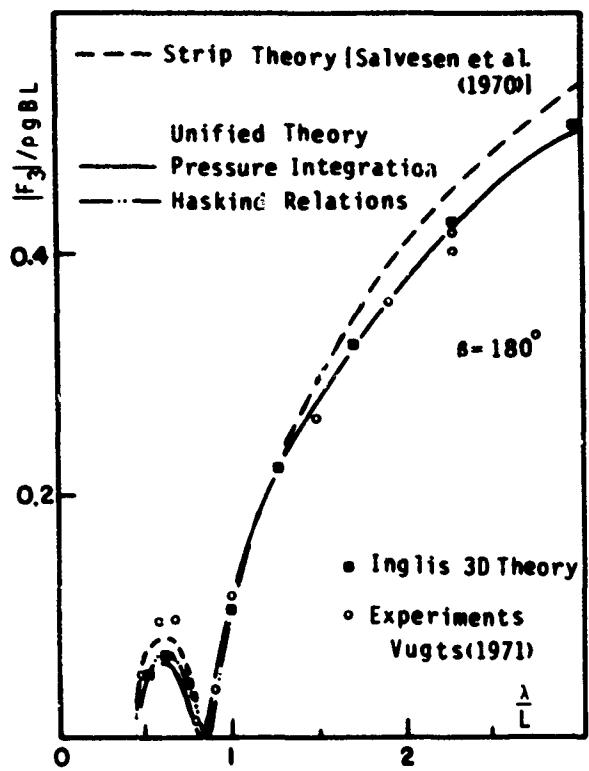


Figure 5 - Heave and pitch exciting force and moment amplitude for a Series 60 hull ( $C_B = 0.7$ ) fixed in head ( $\beta = 180^\circ$ ) and bow ( $\beta = 135^\circ$ ) waves at  $U = 0$ .

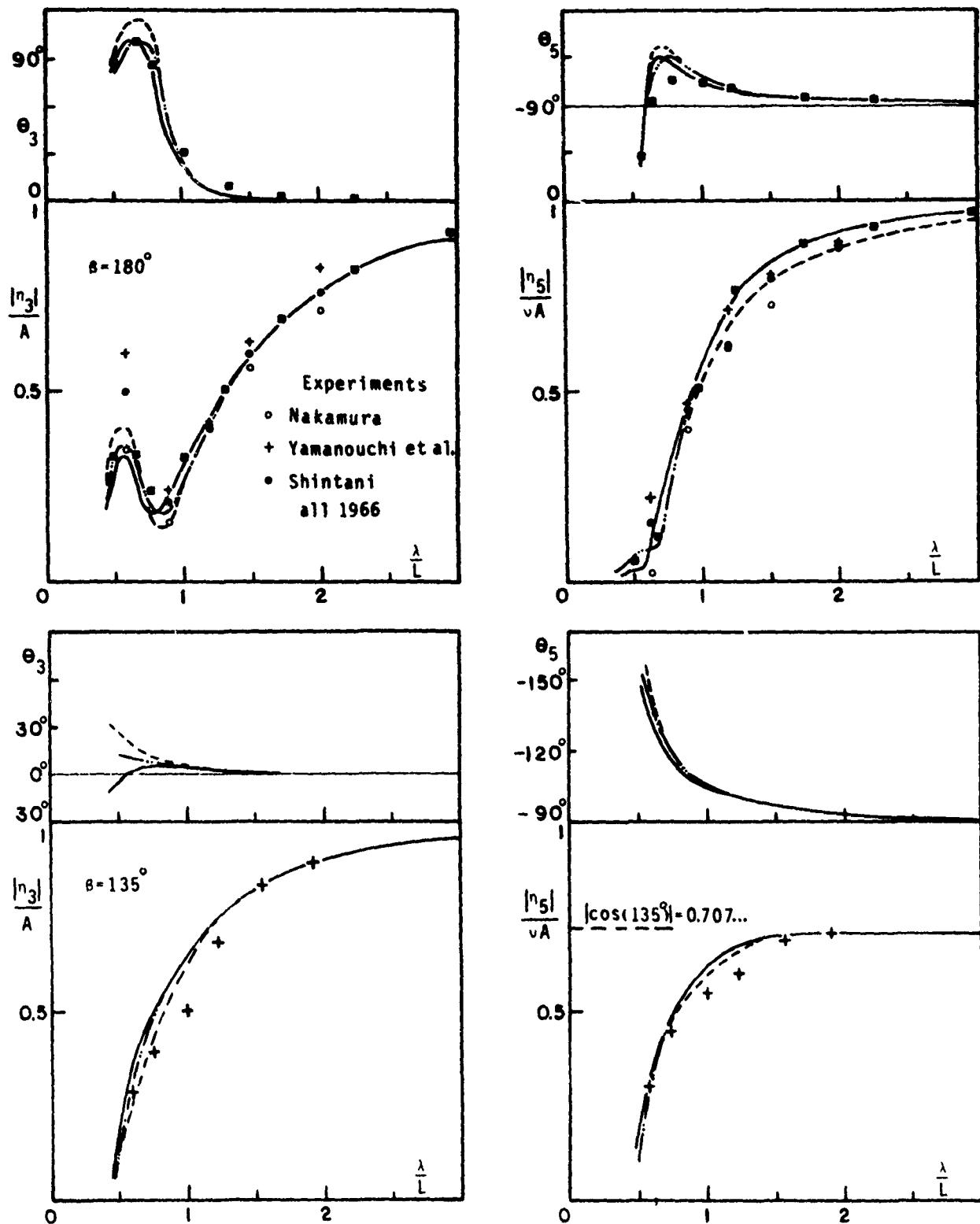


Figure 6 - Heave and pitch motion amplitude and phase of a Series 60 model ( $C_B = 0.7$ ) in head ( $\beta = 180^\circ$ ) and bow ( $\beta = 135^\circ$ ) waves at  $U=0$ .

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A linear theory is presented for the zero-speed heave and pitch motions of a ship in regular free-surface waves, valid for an arbitrary angle of incidence and all wavelengths of practical interest. Slender-body approximations are used to determine the hydrodynamic force distribution for the radiation and the diffraction problems. Computations are shown for a Series 60 hull and comparisons are made with strip theory, a three-dimensional theory and experiments.			